Math 110, Number Systems for Teaching (XM)(4 credits)
Section 1, MTWF 10:20 – 11:20 pm , Swart 302
Spring 2020 — University of Wisconsin Oshkosh

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Office Hours: I am available for drop-in help at my office (Swart 219) as follows:

. Mondays, 1:00 – 2:00 pm
. Wednesdays, 2:00 – 3:00 pm
. Fridays, 2:00 – 3:00 pm

Email me for an appointment if you need to meet outside of these times.

Course Description: Exploring, conjecturing, communicating and reasoning within the content domain of the whole numbers, the integers, the rational numbers and the real numbers. Includes experiences with sets, number sense and numeration, number systems, number theory, concepts of operations on numbers, ratios, proportional reasoning, computational algorithms and estimation. Prerequisite: Math 103, with a grade of C or better or placement.

Core Learning Outcomes: Upon successful completion of this course, students should be able to:

- Create and use a variety of problem solving strategies. These include direct modeling of the physical situation, generating and organizing data, and observing numeric and geometric patterns.
- Communicate the problem-solving process and the resulting mathematics both orally and in writing using the language of mathematicians.
- Distinguish between an explanation of how to solve a problem and an argument that explains why it works.
- Use logic and structure (rather than authority) to determine when a solution is correct and complete.
- Craft arguments using fundamentals of logical reasoning.
- Discuss mathematical activity using the language of doing mathematics (e.g. conjecture, counter example, theorem, proof).
- Refute claims by creating and using counter examples.
- Distinguish between and use both inductive and deductive reasoning.
- Listen, evaluate and respond effectively to the mathematical ideas of peers.
- Build connections among, and work with, a variety of representations.
- Analyze children's mathematical models and explain how these relate to the mathematical work of teaching.
- Experience the beauty and power of mathematics.
- Describe and model the grouping structure that underlies place value number systems.

- Explain and model standard and non-standard arithmetical algorithms.
- Describe connections between arithmetical structure and algebra.
- State and apply the Fundamental Theorem of Arithmetic.
- Use prime factorization to make sense of divisibility, GCF, and LCM.
- Model fractions, ratios, integers, and their operations using sets, areas, and number lines.
- Use ratios, proportions, and percentages to accurately and precisely solve problems and interpret data.
- Explore the relationships among natural numbers, integers, rational numbers, and real number systems.
- State, interpret, and apply key definitions related to properties and operations with numbers and number theory (e.g. number, numeral, digit, integer, rational number, factor, multiple, prime, identity, inverse, commutativity).

Explore Course Connections: This course is an Explore Nature course for the University Studies Program. The Explore classes are designed to provide a solid foundation for your liberal arts education. A liberal arts education provides you with broad knowledge about the world in which we live as well as skills that are transferable across all disciplines. These skills will be especially helpful to you in your future career as a teacher. Specifically, in this course, we will develop communication and problem solving skills in conjunction with the mathematics we learn.

Required Materials: For this course, you will need the following:

Textbook: Beam, J. et.al. (Fall 2019) *Big Ideas in Mathematics for Future Elementary Teachers: Big Ideas in Numbers and Operations.* This text is available at the University Bookstore.

Bridges Account (online): Some of the assignments & activities in the Math 110, 211, & 213 texts will require access to the K-5 Bridges curriculum—an online K-5 mathematics curriculum currently used by the Oshkosh Area School District. You can set up and use a free account (during your classes here) by visiting:
https://bridges.mathlearningcenter.org/user/register/?accountcode=ceuni76158
Once created, you can log into your account at any time by visiting:
https://bridges.mathlearningcenter.org

Canvas Access (online): Canvas will be used for email communication, essential course information, announcements, & assignments (e.g. discussion boards & uploaded documents), so please ensure you have access to Canvas and that your posted email is correct. Limited grade information may periodically be available through Canvas as well.

Microsoft Office: Papers submitted to Canvas must be in Microsoft Word (MSWord) format. Students with a valid UWO email account can install Office 365 (including MSWord) for free by visiting:
https://www.microsoft.com/en-us/education/products/office/default.aspx

Format & Attendance Policy: As a teacher, you will need to think mathematically, express and articulate your thinking & reasoning, and be able to solicit and understand the reasoning of others. To help you develop these abilities, most of class time will be spent working on & discussing interesting problems in small groups and as a class. You are always expected to fully participate in all activities by thinking about the problems, participating in their solutions, and communicating your ideas with others. As explained later, part of your grade is based on your contributions to team and whole-class work & discussions; thus too many absences (whether excused or not) can impact your grade. If you must miss class for any reason, it is your responsibility to inform me in a timely manner and make appropriate arrangements, if possible.

Making Mistakes: Making mistakes is an important (and expected) part of mathematical work and often provides powerful learning experiences, both for yourself and others. I encourage you to share your mistakes as well as your successes with our class during class time. Because mistakes are expected, your grade in this course is not fixed until the end of the semester.

Early Alert: During the 5th week of classes, you may receive an Early Alert email; it is important you read it carefully. Early Alert is a program providing you with an Early Grade Report from faculty. These reports will highlight any academic or attendance issues and indicate resources or steps available to help you improve. This will help you progress early-on, since students are commonly unaware of or over-estimate their performance.

Resources: There are many resources available to help you succeed in this course as well as thrive at UWO. If you are struggling with this course, please talk with your professor during office hours or make an appointment to meet outside of the scheduled times. Additionally, the following places are here to help you:

Center for Academic Resources: The Center for Academic Resources (CAR) provides free tutoring for students in most undergraduate classes on campus. CAR is located in the Student Success Center, Suite 102. Check the Tutor List page on CAR's website (*https://uwosh.edu/car*) for a list of tutors.

Writing Center: The Writing Center provides helpful feedback on your writing assignments. You may schedule an appointment by visiting their website: *https://uwosh.edu/writingcenter* drop-ins are also sometimes available.

Other Resources: Please check out *https://uwosh.edu/resources* for additional resources available to you.

Standards & Evaluation: This will be a standards-based class, meaning that all activities, assignments, and grades aim at providing you with opportunities to develop and demonstrate proficiency in our class' standards. Our class has two types of types of standards (both listed at the end of this syllabus): *practice standards* and *content standards*.

Practice Standards (i.e. *math practices*) describe values, characteristics, habits, and skills that are critical to mathematical thinking. Successful students develop these

math practices through consistent & conscientious actions & choices—usually requiring the breaking of old habits & attitudes.

Content Standards describe the big mathematical ideas and procedures of this class. Successful students are able to recognize, explain, model, and apply these math ideas to new situations, *not* just mimic and repeat memorized procedures.

Activities and assignments (including group activities, class presentations & discussions, reading assignments, and problem sets) will be carefully chosen to help you develop math practices and a deep understanding of the content. Proficiency will be assessed through a diverse set of activities, including peer evaluations, written reflections, class observations, graded problem sets, problem write-ups, quizzes, and a comprehensive final exam (given in class on the last day of class).

Your performance on each standard will be determined as follows:

Practice Standards (except PS4 & PS5): Your performance on the standard will be the average of your performance on each of its benchmarks, equally weighted. Your performance on each benchmark will be the percent (of all the times it was assessed) that you successfully performed it.

Practice Standards 4 & 5 (PS4 & PS5): Your performance on the standard will be the percent (of all the times one of its benchmarks was assessed) that you successfully performed it—so the performance on all benchmarks are pooled together, rather than assessed separately.

Content Standards: Your performance on the standard will be the percent (of all the times a big idea was assessed) that you correctly demonstrated that knowledge—all pooled together, as with PS4.

Grades: Your final grade will be determined by your overall performance, which is the average of your performance on each standard (equally weighted). Cut-offs will be no stricter than:

	Grade +	Grade	Grade -
A*	n/a	93%	90%
B*	87%	83%	80%
C*	77%	73%	n/a
D**	67%	64%	60%

* To earn a C or higher, performance on *all* standards must be 60% or higher.
** To pass with a D- or higher, performance on *all but 2 standards* must be 60% or higher.

Disclaimer: If any substantive changes are made in the course syllabus, notification will be provided in a timely manner and a revised syllabus will be made available.

Academic Integrity: According to university policy, students are responsible for "the honest completion and representation of their work, for the appropriate citation of sources, and for respect of others' academic endeavors." (UWS 14.01, Wisconsin Administrative Code) Taking credit for others' work or presenting others' ideas as your own is also

a form of academic dishonesty. Violations of academic honesty may result in loss of credit for the work in question, failing grade, or even suspension from the University. Be honest in your work AND protect your work from others' dishonesty.

Work Sample Collection: Examining and discussing others' work & solutions (both correct & incorrect) are powerful ways to learn about mathematics & teaching. To provide these opportunities for students & educators, I sometimes collect samples of student work. Collected work might be used in future discussions, assignments, activities, teacher preparation programs, or (with appropriate approval & oversight) research presentations & publications. There will be no compensation for collected work.

Although confidentiality will be maintained by the removal of any identifying information, complete anonymity of *handwritten* work cannot be guaranteed. If you do *not* want your work collected, send me an email with your full name, clearly telling me so. There will be no repercussions for doing so.

A Word of Encouragement: This class is likely to be very different from others you have had. It will be challenging, but I truly believe that everyone is capable of being successful in our class. I recognize that math does not come easily for everyone, but please know that I am here to help you. Please contact me or stop by my office when you have questions, concerns, or just want to discuss your ideas.

Practice Standards & Key Benchmarks
PS1: Make sense of problems and persevere in solving them. A. Identifies, makes, and uses appropriate assumptions B. Explains and describes what would constitute a solution to the problem. C. Expresses and articulates one's own problem solving process, including decisions & approaches taken and their impact. D. Makes (and articulates) intentional and purposeful choices, based on strategy, data, evidence, and reasoning–rather than jumping to conclusions. E. Effectively uses these specific strategies: direct modeling, generating & organizing data, and Observing numeric & geometric patterns. F. Provides a solution that is both correct and complete.
PS2: Generate and utilize visual mathematical representations. A. Creates physical or visual models when appropriate or helpful. B. Captures intended relationships & concepts correctly and accurately. C. Effectively generates, works with, and interprets set models. D. Effectively generates, works with, and interprets Venn diagrams. E. Effectively generates, works with, and interprets base blocks. F. Effectively generates, works with, and interprets area & array models. G. Effectively generates, works with, and interprets number line models. H. Effectively generates, works with, and interprets tape/bar diagrams & ratio tables.
PS3: Reason mathematically and construct viable arguments. A. Uses language that aims to convince & explain *why* it is correct, instead of *how* it was solved. B. Recognizes and appropriately argues situations justifiable by examples or counterexamples and those requiring a general argument. C. Incorporates and draws effectively upon correct assumptions, definitions, and prior results. D. Addresses sufficient criteria to argue the point (even if issues with execution). E. Justifies the solution fully by addressing all important details well enough to convince a reasonable skeptic.
PS4: Generate and utilize verbal and symbolic mathematical representations. A. Uses accurately all conventionally established representations & symbols (e.g. =). B. Performs meaningful algebraic manipulations accurately & without errors. C. Performs accurate calculations & follows key conventions (e.g. order of operations). D. Meaningfully uses and interprets symbols and language associated with operations (addition, subtraction, multiplication, division, and exponents). E. Meaningfully uses & interprets symbols & language associated with fractions & percentages. F. Uses & interprets general math language (e.g. quantifiers) accurately & conventionally. G. Uses descriptive, precise, & developmentally appropriate teacher talk, avoiding non-descriptive jargon (e.g. avoiding the terms carrying, borrowing, & cancelling).
PS5: Understand and critique the reasoning of others. A. Poses questions & problems that target or draw-out specific reasoning or understanding. B. Demonstrates understanding of others' explanations or approaches. C. Recognizes & identifies the correct and incorrect ideas, approaches, & reasoning of others. D. Models & explains interpretations, strategies, & approaches typical of young children. E. Uses language that values & encourages others' valid mathematical ideas & approaches.

Practice Standards & Key Benchmarks continued....

PS6: Contribute to our mathematical communities.

A. Attends class regularly.
B. Comes prepared for class, having completed all required preparations.
C. Collaborates by: listening to, respecting, & considering others' contributions; sharing own thoughts, ideas, & reasoning; asking & answering questions; and being on-task & working well with group members.
D. Engages in whole-class discussion by: presenting problems at the board; asking & answering questions; and listening & responding to peers' questions & ideas.
E. Contributes meaningfully to the class or broader math community through other means.

Content Standards & Big Ideas

CS1: Number Theory

A. Demonstrates (through computation, explanation, & modeling) a correct understanding of the following procedures: factoring numbers; determining if a number is prime, composite, or neither; determining numbers' prime factorization; and computing the least common multiple (LCM) & greatest common factor (GCF) of numbers.
B. Demonstrates (via explanation, correct use of terminology, example/non-example generation) a correct understanding of the following definitions: prime/composite number, relatively prime numbers, factor/divisor of a number, common factor/multiple of some numbers, and multiple of a number.
C. Uses numbers' factors & prime factorizations to identify their properties (e.g. factors, common multiples, common factors).

CS2: Numbers

A. Demonstrate an understanding of our common sets of numbers, including: natural numbers, whole numbers, integers, rational numbers, and irrational numbers.
B. Compare & contrast sets of numbers by properties that they share and do not share (e.g. density, closure & behavior under operations).
C. Demonstrate (by explanation, modeling, and correct symbol usage) an understanding of the concept of additive inverses and how negative numbers and the negation symbol (e.g. $-x$) are defined from this concept.
D. Explain and justify procedures & properties of the arithmetic of positive & negative numbers, using models that can meaningfully distinguish between operations (of addition & subtraction) and number types (positive & negative).

CS3: Place Value and Number Systems

A. Demonstrates (by computation, explanation, & modeling) a correct understanding of the following procedures: express quantities as base n numerals, count in base n, & convert between bases.
B. Demonstrates (by explanation, correct use of terminology, example/non-example generation) a correct understanding of the following definitions: number, numeral, & digit.
C. Determines and compares quantities expressed as base n numerals.
D. Uses the grouping structure of a base n numeral to: understand the relative values of different places in a base n numeral; restructure a number using different base n units; and determine a number's properties (e.g. divisibility by three, odd/even, etc.).

Content Standards & Big Ideas continued....

CS4: Sets & Binary Operations.

A. Demonstrates (by computation, explanation, & modeling) a correct understanding of set relations (e.g. equality, equivalence, subset).
B. Demonstrates (by computation, explanation, & modeling) a correct understanding of set operations (e.g. union, intersection, complement).
C. Demonstrates (by explanation, correct use of terminology, example/non-example generation) a correct understanding of the following definitions: binary operation, associative property, commutative property, identity, inverse, & distributive property.

CS5: Addition & Subtraction.

A. Demonstrates (by computation, explanation, & modeling) a correct understanding of standard and alternative algorithms for addition & subtraction (for all number types), including why common denominators are needed for addition & subtraction of fractions.
B. Demonstrates (by explanation & modeling) a correct understanding of addition & subtraction, along with their properties.
C. Explains & uses the inverse relationships between addition & subtraction.
D. Recognizes, creates, & distinguishes among the different types of contexts that apply to these operations (e.g. CGI problem types).

CS6: Multiplication & Division.

A. Demonstrates (by computation, explanation, & modeling) a correct understanding of standard and alternative algorithms for multiplication & division (for all number types), including why common denominators are *not* needed for multiplication (and sometimes not for division) of fractions.
B. Demonstrates (by explanation & modeling) a correct understanding of multiplication & division, along with their properties.
C. Explains & uses the inverse relationships between multiplication & division.
D. Recognizes, creates, & distinguishes among the different types of contexts that apply to these operations (e.g. CGI problem types).

CS7: Rational Number Concepts & Operations.

A. Demonstrates (by computation, explanation, & modeling) a correct understanding of the following procedures: finding equivalent fractions; diverse strategies for comparing fractions; changing between mixed numbers & improper fractions; finding common numerators and denominators; and ways to convert between fraction & decimal forms.
B. Demonstrates (by explanation, modeling, & comparison) the part-whole meaning of fractions, including: unit fractions and fractions as iterations of unit fractions.
C. Demonstrate (via explanation & procedure) the distinction between rational and irrational numbers both by definition and decimal expansion.

CS8: Ratios & Proportional Reasoning.

A. Demonstrate (through computation, explanation, & modeling) a correct understanding of the following procedures: common procedures for finding equivalent ratios; common procedures for determining if ratios are equivalent; common procedures for solving proportions; ways of expressing a ratio as a part-part (rather than part-whole) fraction or percentage; and ways of converting between different units and rates.
B. Demonstrate (via explanation & modeling) understanding of ratios, percentages, & rates.
C. Demonstrate (by explanation & modeling) an understanding of percentages as relative to some "whole", using the idea of a whole to compare, represent, and use percentages.
D. Recognize, create, & distinguish among proportional & non-proportional situations.