Math 415, Modern Geometry for Teaching(4 credits)
Section 1, TH 1:20 – 3:20 pm , Swart 4
Spring 2020 — University of Wisconsin Oshkosh

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Office Hours: I am available for drop-in help at my office (Swart 219) as follows:
. Mondays, 1:00 – 2:00 pm
. Wednesdays, 2:00 – 3:00 pm
. Fridays, 2:00 – 3:00 pm
Email me for an appointment if you need to meet outside of these times.

Course Description: An intuitive and investigative study of geometry and axiomatics emphasizing modeling, problem solving, and communication. Topics are chosen from axiomatic systems, synthetic (constructive) geometry, transformational (motion) geometry (reflections, rotations, translations, and glide-reflections), analytic (coordinate) geometry, symmetry, fractal geometry, spatial visualization, topology and graph theory. Prerequisites: Mathematics 211 and 213 each with a grade of C or better.

Core Learning Outcomes: Upon successful completion of this course, students should be able to:

- Create and use a variety of problem solving strategies including reasoning from diagrams, and using geometric models to find generalities and constraints.
- Understand the limitation of geometric representations and the distinction between these representations and the ideal objects they represent.
- Communicate mathematics both orally and in writing using the language of mathematicians. Attend carefully to mathematical language distinctions including quantifiers and conditional statements.
- Listen, evaluate and respond effectively to the mathematical ideas of peers.
- Formulate formal mathematical arguments using axioms, definitions, and previously established results, and come to understand, through logic and structure - as opposed to another authority - when a solution is correct and complete.
- Interpret and model axiomatic systems.
- Perform and justify constructions with compass and straight edge.
- Demonstrate a deeper understanding of Euclidean geometry topics taught in middle school such as the Pythagorean Theorem and its converse, analytic geometry techniques, and measurement of area and volume.
- Perform rigid motions and dilations of the plane and justify properties of these transformations.
- Compare and contrast Euclidean and non-Euclidean geometries.
- Articulate ways to foster geometric thinking in children.

Required Materials: For this course, you will need the following:

Textbook: Beam, J. et.al. (Winter 2020) *Big Ideas in Euclidean and Non-Euclidean Geometries*. This text is available at the University Bookstore.

Canvas Access (online): Canvas will be used for email communication, essential course information, announcements, & assignments (e.g. discussion boards & uploaded documents), so please ensure you have access to Canvas and that your posted email is correct. Limited grade information may periodically be available through Canvas as well.

Microsoft Office: Papers submitted to Canvas must be in Microsoft Word (MSWord) format. Students with a valid UWO email account can install Office 365 (including MSWord) for free by visiting:

https://www.microsoft.com/en-us/education/products/office/default.aspx

Construction Tools: From time to time, you will need a ruler, compass, protractor, tape, scissors, stapler, colored markers or pencils, graph paper, tracing paper, and a basic calculator (no cell phones, no graphing or scientific or programmable calculators will be allowed on individual assessments). Bring these to class each time, so that you can access them when you need.

Optional Materials: GeoGebra: GeoGebra (available on most computing and mobile platforms) may be useful for some geometry explorations. GeoGebra is a free program for creating and manipulating dynamic geometric objects and has many other uses as well (see https://www.geogebra.org/ for details).

Euclidea: Euclidea (optional) is a fun and free game available on some mobile devices that is based completely on ruler & compass constructions. Playing it can help you better understand and explore Euclidean geometry constructions.

Format & Attendance Policy: As a teacher, you will need to think mathematically, express and articulate your thinking & reasoning, and be able to solicit and understand the reasoning of others. To help you develop these abilities, most of class time will be spent working on & discussing interesting problems in small groups and as a class. You are always expected to fully participate in all activities by thinking about the problems, participating in their solutions, and communicating your ideas with others. As explained later, part of your grade is based on your contributions to team and whole-class work & discussions; thus too many absences (whether excused or not) can impact your grade. If you must miss class for any reason, it is your responsibility to inform me in a timely manner and make appropriate arrangements, if possible.

Making Mistakes: Making mistakes is an important (and expected) part of mathematical work and often provides powerful learning experiences, both for yourself and others. I encourage you to share your mistakes as well as your successes with our class during class time. Because mistakes are expected, your grade in this course is not fixed until the end of the semester.

Resources: UWO has some resources to help you succeed in this course. If you are struggling, please talk with your professor during office hours or make an appointment to meet outside of the scheduled times. Additionally, the following places are here to help you:

Writing Center: The Writing Center provides helpful feedback on your writing assignments. You may schedule an appointment by visiting their website: *https://uwosh.edu/writingcenter* drop-ins are also sometimes available.

Other Resources: Please check out *https://uwosh.edu/resources* for additional resources available to you.

Standards & Evaluation: This will be a standards-based class, meaning that all activities, assignments, and grades aim at providing you with opportunities to develop and demonstrate proficiency in our class' standards. Our class has two types of types of standards (both listed at the end of this syllabus): *practice standards* and *content standards*.

Practice Standards (i.e. *math practices*) describe values, characteristics, habits, and skills that are critical to mathematical thinking. Successful students develop these *math practices* through consistent & conscientious actions & choices—usually requiring the breaking of old habits & attitudes.

Content Standards describe the big mathematical ideas and procedures of this class. Successful students are able to recognize, explain, model, and apply these math ideas to new situations, *not* just mimic and repeat memorized procedures.

Activities and assignments (including group activities, class presentations & discussions, reading assignments, and problem sets) will be carefully chosen to help you develop math practices and a deep understanding of the content. Proficiency will be assessed through a diverse set of activities, including peer evaluations, written reflections, class observations, graded problem sets, problem write-ups, quizzes, and a comprehensive final exam (given in class on the last day of class).

Your performance on each standard will be determined as follows:

Practice Standards (except PS5 & PS7): Your performance on the standard will be the average of your performance on each of its benchmarks, equally weighted. Your performance on each benchmark will be the percent (of all the times it was assessed) that you successfully performed it.

Practice Standards 5 & 7 (PS5 & PS7): Your performance on the standard will be the percent (of all the times one of its benchmarks was assessed) that you successfully performed it—so the performance on all benchmarks are pooled together, rather than assessed separately.

Content Standards: Your performance on the standard will be the percent (of all the times a big idea was assessed) that you correctly demonstrated that knowledge—all pooled together, as with PS5.

Grades: Your final grade will be determined by your overall performance, which is the average of your performance on each standard (equally weighted). Cut-offs will be no stricter than:

	Grade +	Grade	Grade -
A*	n/a	93%	90%
B*	87%	83%	80%
C*	77%	73%	70
D**	67%	64%	60%

* To earn a C- or higher, performance on *all* standards must be 60% or higher.
** To pass with a D- or higher, performance on *all but 2 standards* must be 60% or higher.

Disclaimer: If any substantive changes are made in the course syllabus, notification will be provided in a timely manner and a revised syllabus will be made available.

Academic Integrity: According to university policy, students are responsible for "the honest completion and representation of their work, for the appropriate citation of sources, and for respect of others' academic endeavors." (UWS 14.01, Wisconsin Administrative Code) Taking credit for others' work or presenting others' ideas as your own is also a form of academic dishonesty. Violations of academic honesty may result in loss of credit for the work in question, failing grade, or even suspension from the University. Be honest in your work AND protect your work from others' dishonesty.

Work Sample Collection: Examining and discussing others' work & solutions (both correct & incorrect) are powerful ways to learn about mathematics & teaching. To provide these opportunities for students & educators, I sometimes collect samples of student work. Collected work might be used in future discussions, assignments, activities, teacher preparation programs, or (with appropriate approval & oversight) research presentations & publications. There will be no compensation for collected work.

Although confidentiality will be maintained by the removal of any identifying information, complete anonymity of *handwritten* work cannot be guaranteed. If you do *not* want your work collected, send me an email with your full name, clearly telling me so. There will be no repercussions for doing so.

A Word of Encouragement: This class is likely to be very different from others you have had. It will be challenging, but I truly believe that everyone is capable of being successful in our class. I recognize that math does not come easily for everyone, but please know that I am here to help you. Please contact me or stop by my office when you have questions, concerns, or just want to discuss your ideas.

Practice Standards & Key Benchmarks

PS1: Make sense of problems and persevere in solving them.

A. Identifies, uses, and explains (as appropriate) a correct and complete understanding of the problem's assumptions and goals.
B. Monitors (and articulates) one's problem solving process, including making intentional and purposeful choices, based on strategy, data, evidence, and reasoning–rather than jumping to unfounded conclusions.
C. Effectively choose, create, and use a variety of problem solving strategies (e.g. direct modeling, generating & organizing data, observing numeric & geometric patterns).
D. Appropriately and effectively uses these specific strategies: using geometric models & reasoning from diagrams.
E. Generates and considers both a sufficient number and a sufficient diversity of examples during inductive work.
F. Provides a solution that is both correct and complete.

PS2: Generate and utilize visual mathematical representations.

A. Creates physical or visual models when appropriate or helpful.
B. Captures intended relationships & concepts correctly and accurately.
C. Uses and works with the representations in valid ways, recognizing the limitations of visual representations.
D. Draws valid and relevant conclusions & interpretations from representations, distinguishing between visual models and the ideal objects they represent.
E. Translate geometric objects (e.g. points, lines, polygons, & conic sections) and situations into algebraic representations (when appropriate), by making assumptions and choosing parameters that are suitable.
F. Follows conventions (when appropriate) when using culturally established representations & symbols (e.g. =).
G. Demonstrate (by explanation & modeling) geometric properties & relationships (e.g. distance, parallelism, perpendicularity, midpoint) and their algebraic counterparts (e.g. slope, distance formula, midpoint formula).
H. Performs meaningful algebraic manipulations accurately & without errors.

PS3: Reason mathematically and construct viable arguments.

A. Uses language that aims to convince & explain *why* it is correct, instead of *how* it was solved.
B. Recognizes and appropriately argues situations justifiable by examples or counterexamples as well as those requiring a general argument.
C. Recognizes and builds arguments upon complete sets of correct assumptions.
D. Uses correct axioms effectively (when appropriate) to strengthen arguments.
E. Uses correct definitions effectively (when appropriate) to strengthen arguments.
F. Uses correct theorems effectively (when appropriate) to strengthen arguments.
G. Addresses a complete and appropriate exit strategy (i.e. sufficient criteria to argue the point).
H. Justifies the solution fully by addressing all important details well enough to convince a reasonable skeptic.

Practice Standards & Key Benchmarks continued....

PS4: Generate and utilize verbal and symbolic mathematical representations.

A. Expresses mathematical ideas and relationships using clear, precise mathematical statements.
B. Follow mathematical conventions when interpreting & using conjunctions (e.g. or, and).
C. Meaningfully creates, uses, and interprets conditional (if-then) statements, their converses, and their contrapositives.
D. Meaningfully and conventionally use & interpret mathematical quantifiers (e.g. at least, at most, for all, there exists, some) and mixed quantifiers.
E. Correctly interpret and express the negation of a statement or condition, particularly non-dichotomous situations, where the negation is not the same as an opposite extreme.
F. Uses & interprets other discipline-specific language and terminology accurately & conventionally.

PS5: Understand and critique the reasoning of others.

A. Poses questions & problems that target or draw-out specific reasoning or understanding.
B. Demonstrates understanding of others' explanations or approaches.
C. Recognizes & identifies the correct and incorrect ideas, approaches, & reasoning of others.
D. Models & explains interpretations, strategies, & approaches typical of young children.
E. Uses language that values & encourages others' valid mathematical ideas & approaches.

PS6: Contribute to our mathematical communities.

A. Attends class regularly.
B. Comes prepared for class, having completed all required preparations.
C. Collaborates by: listening to, respecting, & considering others' contributions; sharing own thoughts, ideas, & reasoning; asking & answering questions; and being on-task & working well with group members.
D. Engages in whole-class discussion by: presenting problems at the board; asking & answering questions; and listening & responding to peers' questions & ideas.
E. Contributes meaningfully to the class or broader math community through other means.

PS7: Work with and within axiom systems.

A. Demonstrate (by explanation & modeling) an ability to understand and interpret the relationships established by a set of axioms.
B. Determine whether a set of axioms is consistent and non-redundant.
C. Work within an axiom system to create or use definitions and to develop & prove mathematical truths (i.e. theorems).
D. Contrast axiom systems in order to determine, explain, & illustrate how changes in the axioms impact the resulting geometry (e.g. theorems & definitions).

Content Standards & Big Ideas

CS1: Euclidean Objects

A. Demonstrates (by modeling, explanation, & correct use of terminology) an understanding of planar objects and their properties, including: points, lines, rays, line segments, angles, polygons (vertices, edges, vertex angles, diagonals, exterior angles, & central angles), and circles (center, radius, chord, diameter, sector, secant, & tangent).
B. Demonstrate (by modeling, explanation, & correct use of terminology) an understanding of spatial objects and their properties, including: polyhedra (faces, edges, vertices), cylinders, cones, & spheres.
C. Use properties to classify angles (acute, right, obtuse), triangles (scalene, isosceles, equilateral, acute, obtuse, right), quadrilaterals (e.g. parallelograms, rectangles, rhombi, trapezoids, & squares), polygons (e.g. concave, convex, regular), polyhedra (regular, prism, pyramid), and other geometric objects.
D. Recognize, model, explain, and use common relationships among objects, such as: lines (parallel, perpendicular, transversal), angles (adjacent, vertical, complementary, supplementary), circles (inscribed & central angle), & nets for polyhedra.
E. Demonstrate (by modeling & explanation) an understanding of the locus definitions and properties of conic sections: circles, ellipses, parabolas, & hyperbolas.
F. Construct & describe special centers for triangles, including the: centroid, orthocenter, incenter, & circumcenter.
G. Determine relationships among the properties of an object.
H. Determine relationships among different classes of objects.

CS2: Euclidean Geometry

A. Demonstrate (by modeling, explanation, & construction) an understanding of Euclid's five postulates (axioms) and their relation to straight-edge & compass constructions.
B. Use straight-edge & compass constructions to make constructable Euclidean objects, such as: perpendicular lines, parallel lines, congruent angles, angle bisectors, & perpendicular bisectors.
C. Create and use accurate nets to represent and work with 3-dimensional objects.
D. Illustrate, explain, and apply the propositions from Euclidean geometry.
E. Demonstrate (by illustration, explanation, & application) an understanding of these specific theorems: Vertical angles theorem, Parallel line theorems (AIA, AEA, Corresponding angles), Triangle congruence theorems (ASA, SAS, SSS, AAS), Isosceles triangle theorems, Corresponding parts of congruent triangles are congruent (CPCTC), and the Pythagorean theorem & its converse.

CS3: Non-Euclidean Geometries

A. Demonstrate (by modeling, explanation, & argumentation) an understanding of the properties of & differences between affine plane geometries and projective plane geometries.
B. Contrast non-Euclidean spaces with Euclidean geometry, explaining & illustrating differences in their properties, axioms, theorems, & applicable definitions—examples may include: identification spaces, spheres, hyperbolic spaces, or fractals.
C. Contrast Taxicab & Euclidean geometry to illustrate & explain how the choice of a metric (i.e. how we measure distance) can impact a geometry's objects & properties.

Content Standards & Big Ideas continued....

CS4: Transformational Geometry

A. Demonstrate (by modeling & explanation) an understanding of rigid motions as actions on the plane or space (as opposed to an object and its image), along with their properties.

B. Use properties and distinguishing features (e.g. fixed points & distances, invariant sets, changes in orientation, parallel motion) to define, classify, compare, and contrast rigid motions (e.g. translations, rotations, reflections, & glide reflections).

C. Demonstrate (by modeling & explanation) an understanding of dilations as actions on the plane or space (as opposed to an object and its scaled image), along with their properties (e.g. angle preserving, closed under composition) and defining features (center of dilation & scale factor).

D. Use composition to establish and recognize relationships between different types of transformations and objects, including how rigid motions generate other rigid motions, how rigid motions define congruence, and how the combination of rigid motions and dilations define similarity.

E. Demonstrate (by modeling & explanation) an understanding of symmetry and its relation to rigid motions.